Stability for discrete velocity models of the extended Boltzmann equation

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Discrete velocity models of the Boltzmann equation have recently become quite popular, both for their fluid dynamics applications and for the mathematical problems they have raised. Among the latter, a significant role is played by stability of equilibria (the so called Maxwellian states) and related relaxation problems. In some recent papers [1] [2] such a question has been addressed for the extended version of the simplest and most ancient discrete models (Carleman, McKean, and Broadwell).

Here, for example, is the six velocity Broadwell model

$$\frac{\partial N_1}{\partial t} + \frac{\partial N_1}{\partial x} = N_3^2 - N_1 N_2 - \varepsilon N_1 + \chi_1 \eta \rho + S_1 \tag{1}$$

$$\frac{\partial N_2}{\partial t} - \frac{\partial N_2}{\partial x} = N_3^2 - N_1 N_2 - \varepsilon N_2 + \chi_2 \eta \rho + S_2 \tag{2}$$

$$\frac{\partial N_3}{\partial t} = -\frac{1}{2}(N_3^2 - N_1 N_2) - \varepsilon N_3 + \chi_3 \eta \rho + S_3 \tag{3}$$

on the line ($-\infty < x < \infty$), where $\rho = N_1 + N_2 + 4N_3$. Parameters $\varepsilon > 0$, $\eta > 0$, $\chi_1 \geq 0$, $\chi_2 \geq 0$, $\chi_3 \geq 0$ are constants such that $\chi_1 + \chi_2 + 4\chi_3 = 1$. $S_i \geq 0$ are sources and $S = S_1 + S_2 + 4S_3 > 0$. ε represents an absorption coefficient, and η takes generation processes (like fission for neutrons, or ionization for electrons) into account. χ_i describe the angular distribution of generated particles. Nonlinear stability of Maxwellian states on the line was shown in [2]. For the Carleman equations this was done in [3]. Conditions under which the Maxwellian states are linearly unstable in a bounded region ($0 < x < 1$) were obtained in [1].

Similar equations may appear when studying transport between cells.

References

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