Solving Polynomial Systems

T. Y. Li
Department of Mathematics
Michigan State University
U.S.A

Numerically solving isolated zeros of polynomial systems in affine space has become increasingly important in applications. Many engineering models, such as power flow problem with PQ-specified buses, the most general six degree of freedom manipulators, and various eigenvalue problems, can be formulated as polynomial systems.

Elimination theory is the classical approach to solving systems of polynomial equations, but its reliance on symbolic manipulation makes it seem unsuitable for all but small problems. Moreover, the method reduces the problem to the ill-conditioned problem of numerically solving a high-degree polynomial equation in one variable.

In this talk, a new approach, developed in the last decade, by using the homotopy continuation method will be surveyed. The method involves first solving a trivial system, and then deforming these solutions along smooth paths to the solutions of the target system. The method has been successfully implemented in solving many polynomial systems, and the amount of computation required to find all solutions can be made roughly proportional to the number of solutions.